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Muon anomalous magnetic moment from effective supersymmetry

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Abstract. We present a detailed analysis on the possible maximal value of the muon $(g-2)_\mu \equiv 2a_\mu$ within the context of effective SUSY models with R parity conservation. First of all, mixing among the second and the third family sleptons can contribute at one loop level to $a_\mu^{\rm SUSY}$ and $\tau \to \mu\gamma$ simultaneously. One finds that $a_\mu^{\rm SUSY}$ can be as large as (10–20) $\times 10^{-10}$ for any $\tan\beta$, imposing an upper limit on the $\tau \to \mu\gamma$ branching ratio. Furthermore, the two loop Barr–Zee type contributions to $a_\mu^{\rm SUSY}$ may be significant for large $\tan\beta$, if a stop is light and μ and A_t are large enough ($\sim O(1)$ TeV). In this case, it is possible to have $a_\mu^{\rm SUSY}$ up to $O(10) \times 10^{-10}$ without conflicting with $\tau \to l\gamma$. We conclude that the possible maximal value for $a_\mu^{\rm SUSY}$ is about $\sim 20 \times 10^{-10}$ for any $\tan\beta$. Therefore the BNL experiment on the muon a_μ can exclude the effective SUSY models only if the measured deviation is larger than $\sim 30 \times 10^{-10}$.

1 Introduction

The anomalous magnetic dipole moment (MDM) of a muon, $a_\mu \equiv (g_\mu - 2)/2$, is one of the best measured quantities. Recently, the Brookhaven E821 collaboration announced new data on the anomalous magnetic moment a_μ [1]:

$$a_\mu^{\rm exp} = (11659202 \pm 14 \pm 6) \times 10^{-10}. \qquad (1)$$

On the other hand, the SM prediction for this quantity has been calculated through five loops in QED and two loops in the electroweak interactions [2]. Using the corrected light–light scattering contribution to the a_μ through pion exchange [3], the difference between the data and the SM prediction is

$$\delta a_\mu \equiv a_\mu^{\rm exp} - a_\mu^{\rm SM} = (26 \pm 16) \times 10^{-10}, \qquad (2)$$

which is only a 1.6σ deviation. Therefore, the present data do not indicate any indirect evidence of new physics at the electroweak scale. However, since the ultimate goal of the BNL experiment is to reduce the experimental error down to $\sim 4 \times 10^{-10}$, δa_μ may provide a useful constraint on various new physics scenarios just around the electroweak scale.

The most promising new physics possibility beyond the SM is the minimal supersymmetric standard model (MSSM) and its various extensions, and the muon $(g-2)_\mu$ was one of the basic observables one considered in various SUSY models [4]. After the BNL data were announced in the year 2001, there appeared a lot of works on the muon $(g-2)$ in the context of SUSY models within the general MSSM (even with R parity violation), minimal SUGRA, gauge mediation, anomaly mediation and gaugino mediation scenarios [5]. The conclusions of these works can be summarized as follows in a model independent manner: it is rather easy to accommodate $\delta a_\mu \sim$ (10–70) $\times 10^{-10}$ in SUSY models, if $\mu \tan\beta$ is relatively large and SUSY particles are not too heavy. Also the sign of $a_\mu^{\rm SUSY}$ is correlated with the sign of the μ parameter.

This general conclusion seems to entail that the so-called effective (or decoupling) SUSY models [6], which are attractive ways to solve the SUSY flavor and CP problems, have serious troubles if it eventually turns out that $\delta a_\mu > 10^{-10}$, since the first/second generation sfermions have to be very heavy ($\sim O(20)$ TeV) and almost degenerate for the squark sector. One way to evade this conclusion within the effective SUSY models is simply to invoke R parity violations in order to explain the muon $(g-2)$ within the effective SUSY models [7]. However, mixing between the staus and the smuons was ignored in [7], which is not a valid assumption in generic effective SUSY models. This mixing arises from mismatches between lepton and slepton mass matrices in the flavor space. The effects of such a mixing among the down squarks and its effects on B physics were discussed in [8] some time ago. Our present work is an analogy of these works within the lepton sector (see also [9]). The flavor mixing between the staus and the smuons that contribute to $a_\mu^{\rm SUSY}$ can also enhance the decay $\tau \to \mu\gamma$, for which there exists a new strong bound from BELLE [10]:

$$B(\tau \to \mu\gamma) < 1.0 \times 10^{-6}.$$

$$M_{\tilde{l}}^2 = \begin{pmatrix} V_L^E M_L^2 V_L^{E\dagger} + m_l^2 + \dfrac{\cos 2\beta}{2}(M_Z^2 - 2M_W^2)\mathbf{1} & -m_l(\mu \tan\beta \mathbf{1} + A_l^*) \\ -m_l(\mu^* \tan\beta \mathbf{1} + A_l) & V_R^E M_E^{2T} V_R^{E\dagger} + m_l^2 - \cos 2\beta M_Z^2 \sin^2\theta_W \mathbf{1} \end{pmatrix}. \tag{6}$$

$$\left(\begin{array}{ccc|ccc} \tilde{m}^2_{LL11} & & & -m_e\mu\tan\beta & & \\ & \tilde{m}^2_{LL22} & \tilde{m}^2_{LL23} & & -m_\mu\mu\tan\beta & \\ & \tilde{m}^2_{LL32} & \tilde{m}^2_{LL33} & & & -m_\tau\mu\tan\beta \\ \hline -m_e\mu\tan\beta & & & \tilde{m}^2_{RR11} & & \\ & -m_\mu\mu\tan\beta & & & \tilde{m}^2_{RR22} & \tilde{m}^2_{RR23} \\ & & -m_\tau\mu\tan\beta & & \tilde{m}^2_{RR32} & \tilde{m}^2_{RR33} \end{array}\right). \tag{7}$$

Thus one has to consider $a_\mu^{\rm SUSY}$ and $\tau \to \mu\gamma$ simultaneously.

In this letter, we present a detailed analysis on the muon $(g-2)_\mu$ in the effective SUSY models with R parity conservation, especially the possible maximal value of $a_\mu^{\rm SUSY}$ in view of the expected new BNL data. Lacking definite effective SUSY models, we will basically perform a numerical analysis in a model independent way, imposing the constraint from the unobserved decay $\tau \to \mu\gamma$. This constraint turns out to be especially strong in the large $\tan\beta$ region. For relatively small $\tan\beta$ (up to $\lesssim 10$), the slepton mixing allows $a_\mu^{\rm SUSY}$ to be as large as $\sim 20\times 10^{-10}$ without having a too large $\tau \to \mu\gamma$, if there is large mixing between the staus and smuons in both chirality sectors (namely, $\tilde{\mu}_L$–$\tilde{\tau}_L$ and $\tilde{\mu}_R$–$\tilde{\tau}_R$ mixings). For larger $\tan\beta > 30$, the constraint from $\tau \to \mu\gamma$ becomes very strong. Still the $a_\mu^{\rm SUSY}$ can be as large as 9×10^{-10} at the one loop level. Furthermore, the Barr–Zee type two loop contribution can enhance the $a_\mu^{\rm SUSY}$ up to $(10–20)\times 10^{-10}$, if A_t and μ are of size $\sim O(1)$ TeV and $\tan\beta$ is large. In short, it is not impossible to have $a_\mu^{\rm SUSY}$ as large as $\sim 20\times 10^{-10}$ regardless of $\tan\beta$ in effective SUSY models. Therefore the BNL experiment on the muon $(g-2)_\mu$ can exclude the effective SUSY models without any ambiguities only if $\delta a_\mu > 30\times 10^{-10}$ within the errors.

2 Definitions

Let us first define the $l_i \to l_j\gamma$ form factors L_{ji} and R_{ji} as follows:

$$\mathcal{L}_{\rm eff}(l_i \to l_j\gamma) = e\frac{m_{l_i}}{2}\bar{l}_j\sigma^{\mu\nu}F_{\mu\nu}\left(L_{ji}P_L + R_{ji}P_R\right)l_i. \tag{3}$$

Then, the muon $(g-2)$ or a_μ is related with $L(R)_{22}$ by

$$a_\mu = \frac{1}{2}(g_\mu - 2) = m_\mu^2\left(L_{22} + R_{22}\right), \tag{4}$$

whereas the decay rate for $l_i \to l_{j\neq i} + \gamma$ is given by

$$\frac{{\rm Br}(l_i \to l_{j\neq i} + \gamma)}{{\rm Br}(l_i \to l_{j\neq i} + \nu_i\overline{\nu}_j)} = \frac{48\pi^3\alpha}{G_{\rm F}^2}\left(|L_{ji}|^2 + |R_{ji}|^2\right). \tag{5}$$

We will calculate L, R's relevant to $a_\mu^{\rm SUSY}$ and $\tau \to \mu\gamma$ in the framework of effective SUSY models. Our notation and conventions follow those of [11].

3 $(g-2)_\mu$ from effective SUSY

The slepton mass matrix in the super-CKM basis is given by (see (6) on top of the page). This matrix is taken to be of the following form (neglecting the trilinear couplings for charged leptons for the time being): (see (7) on top of the page). Since we are looking at a CP-conserving effect, all these mass parameters are assumed to be real. The origin of this kind of mixing may be the form of $M^2_{L,E}$, the soft mass matrices in the flavor basis, or $V^E_{L,R}$, the lepton mixing matrices. We can diagonalize the 2–3 submatrix of the LL sector into a mixing angle θ_L and two mass eigenvalues $\tilde{M}_L^2, \tilde{m}_L^2$ in the limit of no LR mixing:

$$\begin{pmatrix} \tilde{m}^2_{LL22} & \tilde{m}^2_{LL23} \\ \tilde{m}^2_{LL32} & \tilde{m}^2_{LL33} \end{pmatrix} = \begin{pmatrix} \cos\theta_L & \sin\theta_L \\ -\sin\theta_L & \cos\theta_L \end{pmatrix} \begin{pmatrix} \tilde{M}_L^2 & \\ & \tilde{m}_L^2 \end{pmatrix} \times \begin{pmatrix} \cos\theta_L & -\sin\theta_L \\ \sin\theta_L & \cos\theta_L \end{pmatrix}, \tag{8}$$

and likewise for the RR sector. The sneutrino mass matrix with the neutrino masses neglected is

$$M_{\tilde{\nu}}^2 = V_L^\nu M_L^2 V_L^{\nu\dagger} + \frac{\cos 2\beta}{2}M_Z^2\mathbf{1}, \tag{9}$$

and the lightest sneutrino mass is

$$m^2_{\tilde{\nu}_3} = \tilde{m}_L^2 + \cos 2\beta M_W^2, \tag{10}$$

when we also ignore the lepton masses. If V_L^ν is different from V_L^E, $M_{\tilde{\nu}}^2$ is diagonalized by a different unitary matrix than the LL sector of $M_{\tilde{l}}^2$. However, this misalignment is compensated by the MNS matrix at the chargino–lepton–sneutrino vertex, and the chargino amplitudes can be expressed in terms of the slepton mixing angles, θ_L and θ_R, if we ignore neutrino and lepton masses in $M_{\tilde{\nu}}^2$ and $M_{\tilde{l}}^2$.

The question of the sizes of $\tilde{m}^2_{AA33}$ and $\tilde{m}^2_{AA23}$ (with $A = L, R$) is a highly model dependent one, depending on the details of the underlying model and may be closely related with understanding the flavor structures in the MSSM. Note that the SUSY flavor problem is stated in the super-CKM basis as follows: the sfermion mass matrices should be flavor diagonal in this basis and/or the sfermion masses should be almost degenerate. Most effective SUSY models in the literature have hierarchical sfermion mass structures (which are almost diagonal with small mixing angles among the different generations) in the flavor basis, namely M_L^2 and M_E^2 [6]. However, it would not be impossible to construct a model of large flavor mixings in the second and third generation sfermions, especially considering the large mixings in the neutrino sector. In the super-CKM basis, the slepton mass matrices M_L^2 and M_E^2 are multiplied by V_L^E, V_R^E and V_L^ν with $V_{MNS} \equiv V_L^E V_L^{\nu\dagger}$. Because of the large mixings among the three light neutrinos, the resulting slepton mass matrices can have large and comparable elements. (A similar argument may be true for the righthanded slepton sector as well.) This is a source of the large mixings among the sleptons, which can enhance the $a_\mu^{\rm SUSY}$ in the effective SUSY models.

With heavier mass eigenstates decoupling, it is straightforward to show that $a_\mu^{\rm SUSY} = a_\mu^C + a_\mu^N$ is given by

$$\begin{aligned} a_\mu^C &= \frac{2}{(4\pi)^2}\frac{m_\mu^2}{m_{\tilde\nu_3}^2}\sum_j \Bigg[g_2^2|Z_{1j}^+|^2 f_1(x_j) \\ &\quad - \frac{m_{C_j}}{v\cos\beta} g_2 Z_{2j}^- Z_{1j}^+ f_2(x_j)\Bigg]\sin^2\theta_L, \\ a_\mu^N &= \frac{2m_\mu^2}{(4\pi)^2}\sum_j \Bigg[\Bigg(\frac{1}{\sqrt2}(g_1 Z_N^{1j} + g_2 Z_N^{2j})\frac{m_{N_j}}{v\cos\beta} Z_N^{3j} f_4(x_{jL}) \\ &\quad - \frac12 |g_1 Z_N^{1j} + g_2 Z_N^{2j}|^2 f_3(x_{jL})\Bigg)\frac{\sin^2\theta_L}{\tilde m_L^2} \\ &\quad - \Bigg(\sqrt2 g_1 \frac{m_{N_j}}{v\cos\beta} Z_N^{1j} Z_N^{3j} f_4(x_{jR}) + 2g_1^2 |Z_N^{1j}|^2 f_3(x_{jR})\Bigg) \\ &\quad \times \frac{\sin^2\theta_R}{\tilde m_R^2} - g_1 Z_N^{1j}(g_1 Z_N^{1j} + g_2 Z_N^{2j})\frac{m_{N_j}\mu\tan\beta}{\tilde m_L^2 - \tilde m_R^2} \\ &\quad \times \left(\frac{f_4(x_{jL})}{\tilde m_L^2} - \frac{f_4(x_{jR})}{\tilde m_R^2}\right)\frac{m_\tau}{m_\mu}\frac{\sin 2\theta_L \sin 2\theta_R}{4}\Bigg], \end{aligned} \tag{11}$$

where $x_j \equiv m_{C_j}^2/m_{\tilde\nu_3}^2$, $x_{jL(R)} \equiv m_{N_j}^2/\tilde m_{L(R)}^2$, and $v^2 = 2m_Z^2/(g_1^2+g_2^2)$. The loop functions are defined as follows:

$$f_1(x) = \frac{1}{12(x-1)^4}(2 + 3x - 6x^2 + x^3 + 6x\log x), \tag{12}$$

$$f_2(x) = \frac{1}{2(x-1)^3}(3 - 4x + x^2 + 2\log x), \tag{13}$$

$$f_3(x) = \frac{1}{12(x-1)^4}(1 - 6x + 3x^2 + 2x^3 - 6x^2\log x), \tag{14}$$

$$f_4(x) = \frac{1}{2(x-1)^3}(-1 + x^2 - 2x\log x). \tag{15}$$

In the limit of no slepton flavor mixing ($\theta_L = \theta_R = \pi/2$), we have checked that our results reduce to the previous results in the MSSM. Let us note that the neutralino–stau loop contribution to a_μ can be enhanced by m_τ/m_μ if both $\tilde\mu_L$–$\tilde\tau_L$ and $\tilde\mu_R$–$\tilde\tau_R$ mixing are (near) maximal. On the other hand, if the mixing is significant only in one chirality sector (namely, if $\theta_L = 0$ or $\theta_R = 0$), there is no such an enhancement factor, and the resulting $a_\mu^{\rm SUSY}$ will be less than in the case $\theta_L = \theta_R = \pi/4$. This was also noticed in [9].

One can also calculate the amplitude for the decay $\tau \to \mu\gamma$. The coefficients relevant to this process read

$$\begin{aligned} L_{23}^C &= \frac{1}{(4\pi)^2}\frac{1}{m_{\tilde\nu_3}^2}\sum_j \Bigg[g_2^2|Z_{1j}^+|^2 f_1(x_j) \\ &\quad - \frac{m_{C_j}}{v\cos\beta} g_2 Z_{2j}^- Z_{1j}^+ f_2(x_j)\Bigg]\frac{m_\mu}{m_\tau}\frac{\sin 2\theta_L}{2}, \end{aligned} \tag{16}$$

$$\begin{aligned} L_{23}^N &= \frac{1}{(4\pi)^2}\sum_j \Bigg[\Bigg(\frac{1}{\sqrt2}(g_1 Z_N^{1j} + g_2 Z_N^{2j})\frac{m_{N_j}}{v\cos\beta} \\ &\quad \times Z_N^{3j} f_4(x_{jL}) - \frac12 |g_1 Z_N^{1j} + g_2 Z_N^{2j}|^2 f_3(x_{jL})\Bigg) \\ &\quad \times \frac{m_\mu}{m_\tau}\frac{\sin 2\theta_L}{2\tilde m_L^2} \\ &\quad - \Bigg(\sqrt2 g_1 \frac{m_{N_j}}{v\cos\beta} Z_N^{1j} Z_N^{3j} f_4(x_{jR}) + 2g_1^2 |Z_N^{1j}|^2 f_3(x_{jR})\Bigg) \\ &\quad \times \frac{\sin 2\theta_R}{2\tilde m_R^2} - g_1 Z_N^{1j}(g_1 Z_N^{1j} + g_2 Z_N^{2j})\frac{m_{N_j}\mu\tan\beta}{\tilde m_L^2 - \tilde m_R^2} \\ &\quad \times \left(\frac{f_4(x_{jL})}{\tilde m_L^2} - \frac{f_4(x_{jR})}{\tilde m_R^2}\right)\frac{\cos^2\theta_L \sin 2\theta_R}{2}\Bigg], \end{aligned} \tag{17}$$

$$\begin{aligned} R_{23}^{C*} &= \frac{1}{(4\pi)^2}\frac{1}{m_{\tilde\nu_3}^2}\sum_j \Bigg[g_2^2|Z_{1j}^+|^2 f_1(x_j) \\ &\quad - \frac{m_{C_j}}{v\cos\beta} g_2 Z_{2j}^- Z_{1j}^+ f_2(x_j)\Bigg]\frac{\sin 2\theta_L}{2}, \end{aligned} \tag{18}$$

$$\begin{aligned} R_{23}^{N*} &= \frac{1}{(4\pi)^2}\sum_j \Bigg[\Bigg(\frac{1}{\sqrt2}(g_1 Z_N^{1j} + g_2 Z_N^{2j})\frac{m_{N_j}}{v\cos\beta} \\ &\quad \times Z_N^{3j} f_4(x_{jL}) - \frac12 |g_1 Z_N^{1j} + g_2 Z_N^{2j}|^2 f_3(x_{jL})\Bigg)\frac{\sin 2\theta_L}{2\tilde m_L^2} \\ &\quad - \Bigg(\sqrt2 g_1 \frac{m_{N_j}}{v\cos\beta} Z_N^{1j} Z_N^{3j} f_4(x_{jR}) + 2g_1^2 |Z_N^{1j}|^2 f_3(x_{jR})\Bigg) \\ &\quad \times \frac{m_\mu}{m_\tau}\frac{\sin 2\theta_R}{2\tilde m_R^2} - g_1 Z_N^{1j}(g_1 Z_N^{1j} + g_2 Z_N^{2j})\frac{m_{N_j}\mu\tan\beta}{\tilde m_L^2 - \tilde m_R^2} \\ &\quad \times \left(\frac{f_4(x_{jL})}{\tilde m_L^2} - \frac{f_4(x_{jR})}{\tilde m_R^2}\right)\frac{\cos^2\theta_R \sin 2\theta_L}{2}\Bigg]. \end{aligned} \tag{19}$$

In order that our numerical analysis be as model independent as possible, we fixed $\tilde m_{LL22} = \tilde m_{RR22} = 10\,\text{TeV}$, and scanned the following parameter range:

$$\begin{gathered} 2 \le \tan\beta \le 50, \quad 0.2\,\text{TeV} \le \mu, \quad M_2 \le 1\,\text{TeV}, \\ (0.1\,\text{TeV})^2 \le \tilde m_{LL33}^2, \quad \tilde m_{RR33}^2 \le (10\,\text{TeV})^2, \\ -(10\,\text{TeV})^2 \le \tilde m_{LL23}^2, \quad \tilde m_{RR23}^2 \le +(10\,\text{TeV})^2. \end{gathered} \tag{20}$$

Note that the effective SUSY models do not necessarily imply that the slepton mass parameters $\tilde m_{LL33}^2$ and/or $\tilde m_{RR33}^2$ should be (electroweak scale)^2. Since slepton Yukawa couplings are small, their effects on the one loop corrected Higgs mass are negligible. Therefore staus and

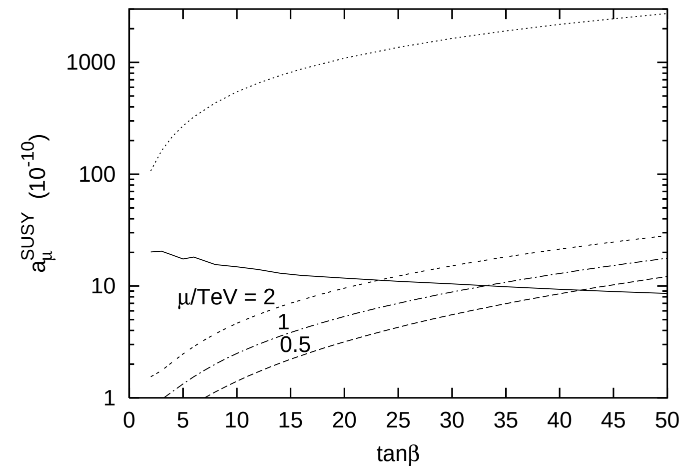


Fig. 1. The possible maximal value of $a_\mu^{\rm SUSY}$ at one loop order in the effective SUSY models as a function of $\tan\beta$, with and without the $\tau \to \mu\gamma$ constraint (the *solid and the dotted curves*, respectively). The lower three curves represent the two loop Barr–Zee type contributions to $a_\mu^{\rm SUSY}$ for $m_S = 100\,\mathrm{GeV}$ and the maximal mixing angle for neutral Higgs bosons

tau sneutrinos need not be light in the effective SUSY models. However the resulting $a_\mu^{\rm SUSY}$ will be very small for very heavy staus and tau sneutrinos. Then we selected parameter sets yielding positive slepton (mass)^2 and satisfying the direct search bounds: $m_{\tilde{\tau}} \geq 85\,\mathrm{GeV}$, $m_{\tilde{\nu}_3} > 44.7\,\mathrm{GeV}$ and $m_{\chi^+} > 103.5\,\mathrm{GeV}$ [12]. We used the GUT relation $M_1/M_2 = 5\alpha_1/3\alpha_2$ to fix M_1 for a given M_2. The trilinear couplings for the charged leptons are set to zero. For large $\tan\beta$, the trilinear couplings are almost irrelevant. For small and moderate $\tan\beta$, it changes the LR mixing parameters, and we have checked that the constrained maximal $a_\mu^{\rm SUSY}$ can change up to $\pm 2 \times 10^{-10}$ when we varied the A_l's from -1 TeV to $+1$ TeV.

In Fig. 1, we show the possible maximal value of $a_\mu^{\rm SUSY}$ (at one loop level) as a function of $\tan\beta$ with and without the $\tau \to \mu\gamma$ constraint in solid and dotted curves, respectively. If $\tan\beta$ is not too large, the $\tau \to \mu\gamma$ constraint does not overkill the $a_\mu^{\rm SUSY}$. For large $\tan\beta$, the one loop contribution to $a_\mu^{\rm SUSY}$ can be much larger, but is strongly constrained by $\tau \to \mu\gamma$. Still the resulting $a_\mu^{\rm SUSY}$ can be as large as 9×10^{-10}. This point is also illustrated by Fig. 2a,b, where we show the region plots for $\tan\beta = 3$ (a) and $\tan\beta = 30$ (b). In Fig. 2a, $a_\mu^{\rm SUSY}$ can reach $O(20 \times 10^{-10})$ for $\tan\beta = 3$, still satisfying the $\tau \to \mu\gamma$ constraint. For $\tan\beta = 30$, we have $a_\mu^{\rm SUSY} \lesssim 10 \times 10^{-10}$ [Fig. 2b]. This behavior can be easily understood, since $a_\mu^{\rm SUSY} \propto \tan\beta$ whereas $B(\tau \to \mu\gamma) \propto \tan^2\beta$. Therefore the constraint becomes much more severe when $\tan\beta$ is large, in which case the one loop $a_\mu^{\rm SUSY}$ is essentially smaller than 10×10^{-10}. The possible maximal value for $a_\mu^{\rm SUSY}$ will decrease as the upper limit on $\mathrm{Br}(\tau \to \mu\gamma)$ gets improved.

In the effective SUSY models, the two loop contributions to the EDM's and MDM's through a third (s)fermion loop could be substantial for large $\tan\beta$ [13,14]. Since the previous discussion implies that the one loop contribution to $a_\mu^{\rm SUSY}$ cannot be larger than $\sim 10 \times 10^{-10}$ for large $\tan\beta$ in the effective SUSY models, it is important to estimate these two loop contributions which may dominate in the large $\tan\beta$ region. The basic formulae for these contributions have been derived both for the neutral and the charged Higgs exchanges with (s)top and/or (s)bottom loops:

$$a_\mu^{\text{two loop}} = -\frac{\alpha}{2\pi}\left(\frac{G_{\rm F} m_\mu^2}{4\sqrt{2}\pi^2}\right)\lambda_\mu^S \sum_{\tilde{f}} N_c^{\tilde{f}} Q_{\tilde{f}}^2 \frac{\lambda_{\tilde{f}}}{m_S^2}\mathcal{F}(m_{\tilde{f}}^2/m_S^2), \tag{21}$$

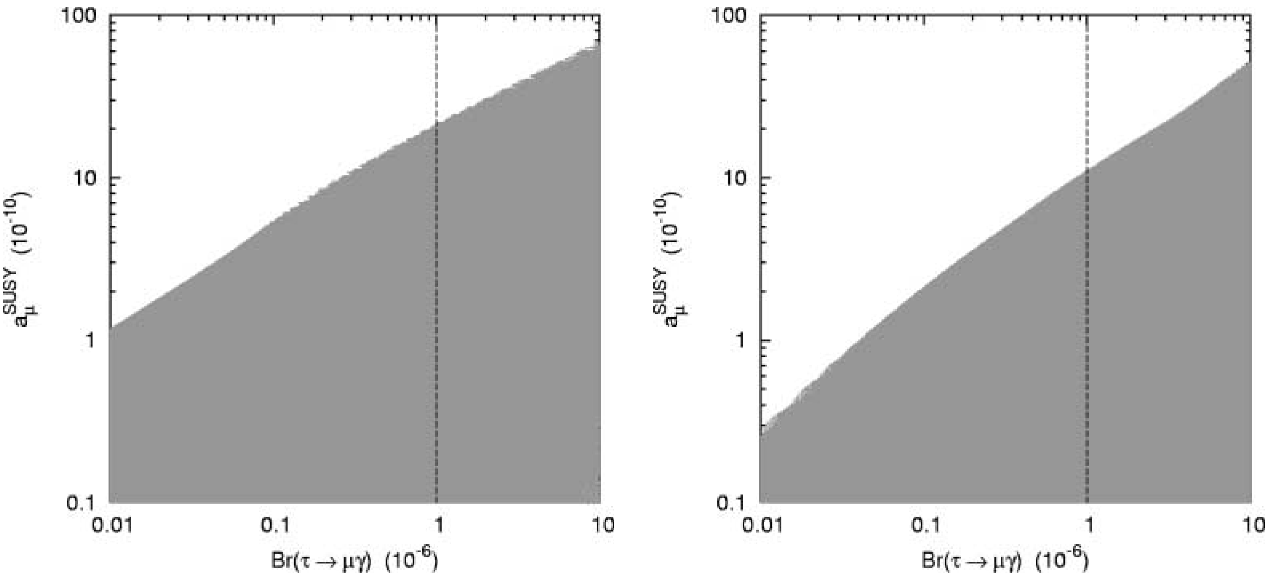
where $N_c^{\tilde{f}}$, $Q_{\tilde{f}}$ and $m_{\tilde{f}}$ are the number of colors, the electric charge and the mass of the internal sfermion in the loop, and m_S (with $S = h^0$ or H^0) is the mass of the exchanged scalar Higgs h^0 or H^0. $\lambda_\mu^{(h^0,H^0)} = (-\sin\alpha, \cos\alpha)/\cos\beta$, where α is the mixing angle of neutral CP-even Higgs bosons. The explicit form of the loop function $\mathcal{F}(z)$ can be found in [14]. Note that the expression in the parentheses,

$$\frac{G_{\rm F} m_\mu^2}{4\sqrt{2}\pi^2} = 23.3 \times 10^{-10},$$

is the size of the SM electroweak corrections to the muon $(g-2)_\mu$, and thus the above two loop Barr–Zee type contributions to the muon $(g-2)_\mu$ can be substantial for large $\tan\beta$, and the large positive μ or the large negative A_f. The larger the trilinear coupling A_t is, the larger $(g-2)_\mu$ one can afford.

In Fig. 1, we also show the two loop Barr–Zee type contribution to $a_\mu^{\rm SUSY}$ for the three different possibilities $\mu = 0.5, 1$ and 2 TeV (the long-dashed, the dot-dashed and the short-dashed curves, respectively). We have assumed the maximal mixing angle for neutral Higgs bosons, and we set $m_S = 100\,\mathrm{GeV}$ (just above the current lower limit on the CP-even heavier neutral Higgs boson H) in order to maximize the desired effect. There is clear evidence that these two loop effects become important as $\tan\beta$ grows. Adding the two loop Barr–Zee type contribution to the one loop effects, the possible maximal value for $a_\mu^{\rm SUSY}$ can easily extend to (20–30) $\times 10^{-10}$ even for large $\tan\beta$. Therefore it would not be possible to completely rule out the effective SUSY models from the BNL experiment on the muon MDM, unless the deviation between the SM prediction and the data is larger than, say, $\sim 30 \times 10^{-10}$.

We also plot the dependence of the possible maximal value of $a_\mu^{\rm SUSY}$ on the SUSY breaking parameter $\tilde{m}_{LL33} = \tilde{m}_{RR33} = \tilde{m}_{33}$ in Fig. 3 for $\tan\beta = 3, 10$ and 40, respectively. The lower (the upper) curves are with (without) the $\tau \to \mu\gamma$ constraint. A larger value of $a_\mu^{\rm SUSY}$ is possible, if $\tilde{m}_{33}$ becomes larger. The reason lies in that in this case one needs a large mixing $\tilde{m}_{LL23}^2$ and $\tilde{m}_{RR23}^2$ in order to have light stops at the electroweak scale if $\tilde{m}_{33}$ becomes large. (Note that we had fixed $\tilde{m}_{LL22} = \tilde{m}_{RR22} = 10\,\mathrm{TeV}$ and we need light stops around a few hundred GeV's in order to have a significant effect on the muon $(g-2)$.) Therefore $a_\mu^{\rm SUSY}$ in the effective SUSY models



a $\tan\beta = 3$ **b** $\tan\beta = 30$

Fig. 2. Regions on the $a_\mu^{\rm SUSY}$–Br$(\tau \to \mu\gamma)$ plane swept as the parameters are varied within the range (20) with $\tan\beta$ fixed at 3 and 30. The vertical dashed line shows the upper bound on the branching ratio at the 90% confidence level

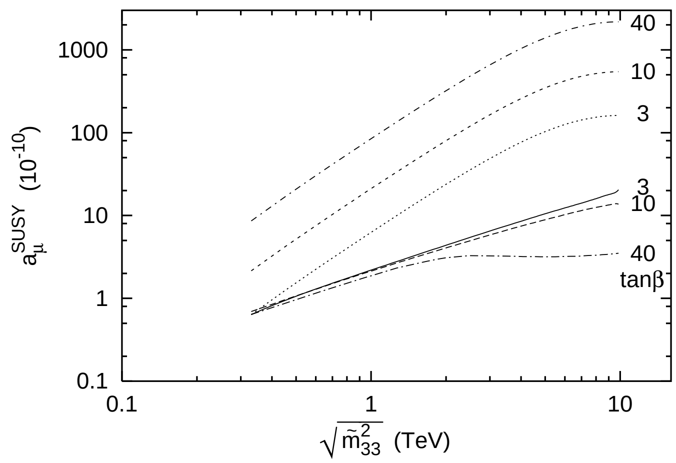


Fig. 3. The possible maximal value of $a_\mu^{\rm SUSY}$ as a function of $\tilde{m}_{33} = \tilde{m}_{LL33} = \tilde{m}_{RR33}$, with and without the $\tau \to \mu\gamma$ constraint (the lower and the upper curves, respectively)

can be $\sim 20 \times 10^{-10}$ at one loop level, if $\tan\beta$ is not too large and the slepton mass parameters involving the 3rd generations are also very large (up to O(few–10) TeV) so that one can have light slepton spectra and large mixings. On the other hand, if one naively applies the idea of light staus directly to the mass parameters $\tilde{m}^2_{LL33}$ and $\tilde{m}^2_{RR33}$ (and necessarily with small flavor mixings $\tilde{m}^2_{LL23}$ and $\tilde{m}^2_{RR23}$ in order to have light but non-tachyonic stops), the resulting $a_\mu^{\rm SUSY}$ cannot be large: $a_\mu^{\rm SUSY} \lesssim 3 \times 10^{-10}$ if $\tilde{m}_{LL33} = \tilde{m}_{RR33} < O(1)$ TeV, for example (see Fig. 3).

Note that the maximum of the $\tan\beta = 40$ curve in Fig. 3 is lower than the $\tan\beta = 40$ point of Fig. 1 about 10×10^{-10}. This is because $\tilde{m}_{LL33}$ and $\tilde{m}_{RR33}$ were assumed to be equal in Fig. 3, but not in Fig. 1. It turns out that in the small $\tan\beta$ case, $a_\mu^{\rm SUSY}$ gets maximized when $\tilde{m}_L^2 \simeq \tilde{m}_R^2$ and $\theta_L \simeq \theta_R \simeq \pi/4$, while in the large $\tan\beta$ case, $\tilde{m}_L^2/\tilde{m}_R^2 \simeq 60$ and $\theta_L \simeq 0.18, \theta_R \simeq \pi/4$. Let us note another point here. The result that $a_\mu^{\rm SUSY}$ reaches 9×10^{-10} when $\tan\beta = 40$, was obtained from (11)–(19). If we treat the LR mixing by fully diagonalizing the 4×4 mass matrix, this maximal number gets reduced to 6×10^{-10}.

4 Conclusions

In conclusion, we considered the muon $(g-2)_\mu$ within the effective SUSY models. In this case, the smuon and the muon sneutrino loop contributions to the muon $(g-2)_\mu$ are negligible. However, the staus can contribute to the muon $(g-2)_\mu$ through the flavor mixing in the slepton sector. Including the current constraint from $\tau \to \mu\gamma$, we find that $a_\mu^{\rm SUSY}$ in the effective SUSY model can be as large as $\sim 20 \times 10^{-10}$ in a reasonable region of parameter space. This bound is fairly model independent within the effective SUSY models and will become smaller once the upper bounds on $\tau \to \mu\gamma$ are improved. Our study shows that $a_\mu^{\rm SUSY}$ can be as large as $\sim 20 \times 10^{-10}$ in the effective SUSY models for all $\tan\beta$ if there is a large mixing between the second and third generation sfermions. For large $\tan\beta$, the constraint from $\tau \to \mu\gamma$ is very strong but $a_\mu^{\rm SUSY}$ can be as large as 9×10^{-10}. Also it can receive additional contributions from two loop Barr–Zee type contributions of similar size. Overall, the possible maximal value for $a_\mu^{\rm SUSY}$ is about 20×10^{-10} so that the BNL experiment on the muon $(g-2)_\mu$ can exclude the effective SUSY models only if the measured deviation is larger than $\sim 30 \times 10^{-10}$.

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